Methods in Philosophical and Critical Thinking

Instructor:
Carlo Martini

T.A.:
Rami Koskinen

04.FORMAL LOGIC (predicate calculus)

Logic Form of arguments

- If he is at home, his hat will be in the hall; his hat is not in the hall; therefore he is not at home.
- If Napoleon was Chinese, he was Asiatic; he was not Asiatic; so he was not Chinese.
- Tweety is a robin; no robins are migrants; therefore Tweety is not a migrant.
- Oxygen is an element; no elements are molecular; therefore oxygen is not molecular.
- Porvoo is between Helsinki and Kuovola; Everything that is between Helsinki and Kuovola is west of Kuovola; Porvoo is west of Kuovola.

All, none, some…

Every, all (etc.) ”Everyone is beautiful”

Some, someone (etc.) ”Someone is wise”

- Someone is wise.
- There exists at least one person that is wise
- There exists at least one person such that that person is wise

- There exists at least one x such that x is wise
- $\exists(x)\ Wx$

Logic of Propositions and Logic of Predicates

- Predicate are properties: e.g. Napoleon was Chinese (abb. ‘Cn’), Napoleon is Asiatic (abb. ‘An’), Napoleon is not Asiatic (abb. ‘-An’), Tweety is a robin (abb. ‘Rt’), Oxygen is not molecular (abb. ‘Mo’).
- Quantifiers:
  - everything
  - nothing
  - some
- There can be predicates that take more than one place: how do we translate sentences such as “Helsinki is between Turku and Porvoo”?

The Elements of Predicate Logic (the predicate calculus)

- The Alphabet of predicate logic consists of:
  - Constants c_0, c_1, c_2, . . .
  - Predicate symbols P_0,P_1,P_2 …
  - Variables: x_0, x_1…, x_2

- Logical constants: connectives + quantifiers ∀ and ∃ ("universal quantifier" and " existential quantifier")

The Elements of Predicate Logic (the predicate calculus)

- 1. Individual constants — a, b, c, …, a_1, a_2, …, a_n, …
  - Refer to particular individuals, names them
  - Correspond roughly to proper nouns in natural language (eg. Socrates, Pekka, Paris, Titanic, Mt. Blanc) and to other determinate particulars (eg. “this rose”, “that dog”)

- 2. Predicates — P, Q, R, S, ... , P_1, P_2, …, P_n, … R_1, R_2, ..., R_n, …
  - Apply to individuals; are true of some individuals.
  - Individuals can instantiate predicates
  - Correspond to properties that individuals can have, eg. ”red”, ”wise”, or to relations between individuals such as ”… loves ….”, ”… is the father of …”
  - If an individual, named by a, instantiates predicate P (that is P is true of a, “a is P”) we write P(a).

The Elements of Predicate Logic (the predicate calculus)

- 3. Variables x, y, z, w, ... x1, x2, …
  - Vary in their interpretation (but is still always some constant)
  - Do not refer to any particular individual
  - Variables "mimic" the role of constants. They "reserve" a place for a constant in predicates: P(x), Q(z), R(x, y) …

- 4. Quantifiers
  - The name 'quantifier' refers to quantities, amounts. In logic there are two quantifiers:

  - Every, all: ∀ universal quantifier
  - Some (there exists): ∃ existential quantifier

Examples

Example: We can express the atomic sentence ”Socrates is human” as:

“Socrates” = a
“human” = P

“Socrates is human” = P(a)

If Q = “mortal”, ”Socrates is mortal” = Q(a)

“If Socrates is human, then Socrates is mortal” : P(a) —> Q(a)

Further: “Plato” = b and R(x, y) is interpreted as “x is y's teacher”,
R(a, b) = “Socrates is Plato’s teacher”

Examples

Example: ”The exists at least one x such that...” => ∃x

Also: ”For every x … ” => ∀x

Correspondingly: ∃y, ∀y, ∃z, …

Examples

Expressing states of affairs in predicate logic

"Someone is wise" $\exists x\, P(x)$

"Everyone is mortal" $\forall x\, Q(x)$

"Every human is mortal" :

For every x, IF (x is human) THEN (x is mortal)

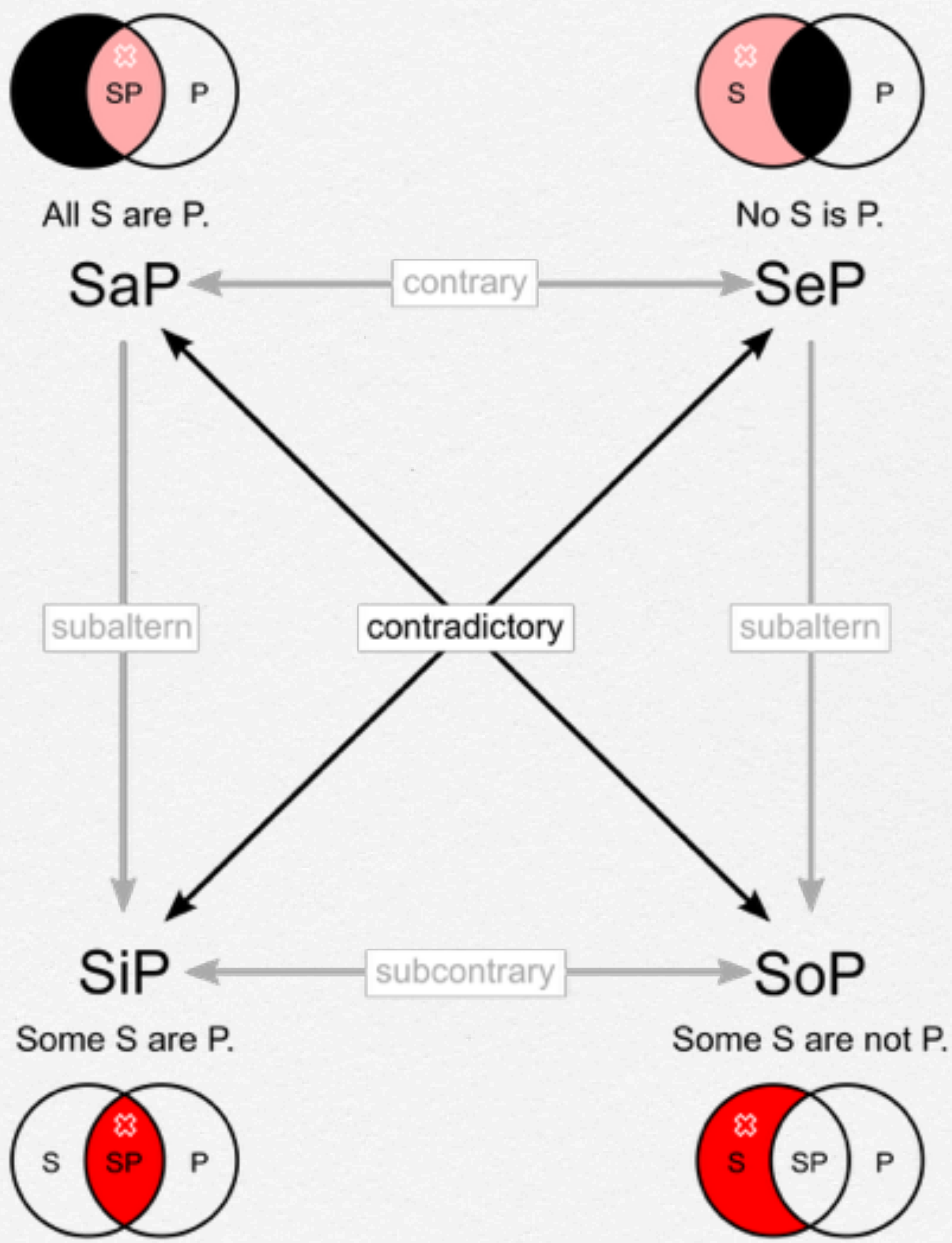
$\forall x\, (P(x) \longrightarrow Q(x))$

"Some talented people are lazy":

There exists at least one x such that: x is talented AND x is lazy.

$\exists x\, (P(x) \,\&\, Q(x))$

Relations between quantifiers



From logics to mathematics

- Example: Universe with n black and n white balls. What sort of claims can we make?
- One can use identity to express amount, but it is inconvenient.
- But we can not say,
  - There are an even number of black balls
  - There are more white balls than black balls
  - The number of balls is finite
  - There is one more black ball than white balls
- How do we express the proposition that “2 + 2 = 4”

Peano's axioms

- (x) S(x) ≠ 0 *For all x, the successor of x is not 0*
- (x) ((y) S(x) == S(y) —> x == y) *For all x, it hold that, for all y, if the successor of x is equal to the successor of y, then x is equal to y*
- (x) x + 0 == x *For all x, x + 0 equals x*
- (x) ((y) x + S(y) == S (x + y))
- (x) x0 == 0
- (x) ((y) xS(y) == yx + x)
- A (0) —> (((x) A(x) —> A(Sx))) —> (y) A(y)

2 + 2 = 4

2 + 2 = 4

S(1) + S(1) = 4

S(S(0)) + S(S(0)) = 4

S [S(S(0)) + S(0)] = 4

S { S [S(S(0)) + 0] } = 4

S (S (S (S(0)))) = 4

4 = 4

Expressive Power of Basic Logic

- Sentential logic (decidable, complete)
- Modal Logic (decidable, complete)
- 1st order predicate logic (semi-decidable, complete)
- 2nd order predicate logic (undecidable, incomplete)
- Natural language (a mess)

- !! Balance between expressive power and usability

Logic and Language

- What is the goal of logics?
- Why is an inference/argument valid?
- Why is an inference/argument invalid?
- Is validity:
  - Language relative?
  - Culture relative?
  - Universal?