Probability and probabilistic fallacies

1 Bayes theorem, theory and applications

1.1 Bayes Theorem

Prove Bayes Theorem starting from the fundamental probability axioms and the definition of conditional probability as discussed in class.

1.2 Application

Consider the following problem:

You have been called to jury duty in a town where there are two taxi companies, Green Cabs Ltd. and Blue Taxi Inc. Blue Taxi uses cars painted blue; Green Cabs uses green cars. Green Cabs dominates the market, with 85% of the taxis on the road. On a misty winter night a taxi sideswiped another car and drove off. A witness says it was a blue cab.

The witness is tested under conditions like those on the night of the accident, and 80% of the time she correctly reports the color of the cab that is seen. That is, regardless of whether she is shown a blue or a green cab in misty evening light, she gets the color right 80% of the time.

You conclude, on the basis of this information:

(a) The probability that the sideswiper was blue is 0.8.

(b) It is more likely that the sideswiper was blue, but the probability is less than 0.8.

(c) It is just as probable that the sideswiper was green as that it was blue.

(d) It is more likely than not that the sideswiper was green.

2 Fallacies

2.1 Linda the bank teller

Consider the following problem:

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable? Explain why.

(a) Linda is a bank teller

(b) Linda is a bank teller and is active in the feminist movement

2.2 Medical diagnoses

Consider the following problem:

"One in a thousand people has a prevalence for a particular heart disease. There is a test to detect this disease. The test is 100% accurate for people who have the disease and is 95% accurate for those who don't (this means that 5% of people who do not have the disease will be wrongly diagnosed as having it). If a randomly selected person tests positive what is the probability that the person actually has the disease?"

This question was put to 60 students and staff at Harvard Medical School. Almost half gave the response 95%. The 'average' answer was 56%. In fact, the correct answer is very different and was given by just 11 participants.

What is the correct answer? Why?