Exercises 02 - Proofs

A summary of the rules of inference is available in: Lemmon, E.J. *Beginning Logic*, CRC Press (1998[1965]): pp. 39-40. In the proofs, note that the first column contains the assumption on which the line is dependent: only lines that are *assumed* (= A, in the right-hand column) appear here. The center column contains the result of applying a *rule of inference* to one or more of the previous lines, each of these lines is numbered progressively, to mark the *length* of a proof. The right hand column is explicative: it indicates the numbers of those lines on which the *rule of inference* is applied, and which *rule of inference* has been used to obtain that line. *Every* line in a proof is obtained either by assumption (A, marked on the right-hand column) or by application of one of the *rules of inference* from one or more of the previous lines.

1 Translate and prove

[Copi, Cohen and McMahon, p. 340] Translate the following verbal sentence into formal propositional logic, using the notation that is the most familiar to you, and prove the inference using both truth tables and natural deduction: "If Denmark refuses to join the European Community, then, if Estonia remains in the Russian sphere of influence, then Finland will reject a free-trade policy. Estonia will remain in the Russian sphere of influence. So if Denmark refuses to join the European Community, then Finland will reject a free-trade policy."

2 Notable theorems

In the following you shall provide proofs for a number of notable theorems. Prove all of the following theorems using natural deduction.

2.1 Three laws of thought — see Copi-Cohen, pp. 351-352

2.1.1 Principle of identity — *if* P, *then* P

$\vdash P \rightarrow P$

2.1.2 Principle of noncontradiction — *not*-(P *and* *not*-P)

$\vdash \sim (P \wedge \sim P)$

2.1.3 Principle of excluded middle — P *or* *not*-P

$\vdash P \vee \sim P$

2.2 De Morgan Theorems

2.2.1 De Morgan 1 — from disjunction negation to negated conjuncts

$\sim (P \vee Q) \vdash \sim P \wedge \sim Q$

2.2.2 De Morgan 2 — from negated conjunction to negated disjuncts

$\sim (P \wedge Q) \vdash \sim P \vee \sim Q$

2.3 Implication theorems

2.3.1 Implication 1 — from implicature to conjuncts

$P \rightarrow Q \vdash \sim (P \wedge \sim Q)$

2.3.2 Implication 2 — falsification of hypotheses

$P \rightarrow Q, \sim Q \vdash \sim P$